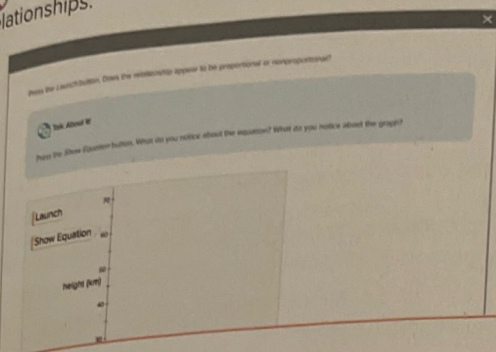


Equations of Proportional Relationships

I Can... write equations to represent proportional relationships and identify the constant of proportionality in the equation representing a proportional relationship.

Explore Proportional Relationships and Equations

Online Activity You will explore the equations of proportional relationships.



Learn Identify the Constant of Proportionality in Equations

Two quantities are proportional if the ratios comparing them are equivalent. This ratio is called the constant of proportionality. Proportional relationships can be represented by an equation in the form $y = kx$, where k is the constant of proportionality.

Words

A linear relationship is proportional when the ratio of y to x is a constant k .

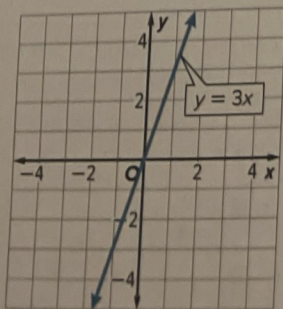
Symbols

$$y = kx, \text{ where } k \neq 0$$

Example

$$y = 3x$$

Graph



Talk About It!

Can you write an equation in the form $y = kx$ for a line that does not pass through the origin? Explain.

Think About It!

How would you begin solving the problem?

Talk About It!

How much would Olivia pay for 10 yogurts at this same rate?

Example 1 Identify the Constant of Proportionality in Equations

Olivia bought six containers of yogurt for \$7.68. The equation $y = 1.28x$ can be used to represent this situation, where y represents the total cost of the yogurt and x represents the number of containers bought.

Identify the constant of proportionality. Then explain what it represents.

Part A Identify the constant of proportionality. Compare the two equations.

$y = kx$, where k is the constant of proportionality

$$y = 1.28x$$

In the equation for the cost of the yogurt, the constant of proportionality k is 1.28.

Part B Explain the constant of proportionality.

The constant of proportionality has the same value as the unit rate. The constant of proportionality means that the cost of each container of yogurt is \$ 1.28. So, the constant of proportionality is 1.28 and the unit rate is \$1.28 per container.

Check

An airplane travels 780 miles in 4 hours. The equation $y = 195x$ models this situation.

Part A What is the constant of proportionality?

$$\begin{array}{l} y \leftarrow 780 \\ x \leftarrow 4 \end{array} \quad \begin{array}{l} \uparrow \\ 195 \end{array}$$

Part B What does the constant of proportionality represent in the context of the problem?

- (A) An airplane travels 390 miles per hour.
- (B) An airplane travels 1 mile per 4 hours.
- (C) An airplane travels 1 mile per 585 hours.
- (D) An airplane travels 195 miles per hour.

 **Go Online** You can complete an Extra Example online.

Learn Proportional Relationships and Equations

You can use the constant of proportionality, or unit rate, to write an equation in the form $y = kx$ that represents a proportional relationship.

You can find k by writing the ratio comparing y to x . To write the ratio, solve the equation $y = kx$ for k .

$$y = kx \quad \text{Write the equation.}$$

$$\frac{y}{x} = \frac{kx}{x} \quad \text{Divide each side by } x.$$

$$\frac{y}{x} = k \quad \text{Simplify.}$$

The constant of proportionality is the ratio $\frac{y}{x}$, where $x \neq 0$.

Example 2 Proportional Relationships and Equations

Jaycee bought 8 gallons of gas for \$31.12.

Write an equation relating the total cost y to the number of gallons of gas x if it is a proportional relationship.

Step 1 Find the constant of proportionality between cost and gallons.

$$\frac{\text{total cost (\$)}}{\text{number of gallons}} = \frac{31.12}{8}$$
$$= \frac{3.89}{1}$$

The constant of proportionality is 3.89

Step 2 Write the equation.

The total cost is \$3.89 multiplied by the number of gallons.

Let y = total cost and x = number of gallons.

So, the total cost for any number of gallons can be found using the equation $y = 3.89x$.

Check

A mechanic charges \$270 for 5 hours of work on a car. Write an equation that compares the total cost to the number of hours he works on a car if it is a proportional relationship.

$$y = kx \quad y = \$ \quad x = h$$
$$k = \frac{270}{5} = 54$$

Talk About It!

What is the equation for a proportional relationship where the constant of proportionality is 3.25?

$$\begin{array}{r} 54 \\ 5 \overline{) 270} \\ \underline{25} \\ 20 \end{array}$$

Talk About It!

How can you use the equation $y = 3.89x$ to determine the cost for 15 gallons of gas?

Go Online You can complete an Extra Example online.

Think About It!
What do you need to find before you can write the equation?

Talk About It!
Suppose you know the perimeter of a square is 52 inches. Explain how you could use the equation $y = 4x$ to find the length of the side length of the square. Then find the length of the side.

gallon min
500 5
100 ← 1

Example 3 Proportional Relationships and Equations

The perimeter of a square is proportional to the length of one side. A square with 10-inch sides has a perimeter of 40 inches.

Write an equation relating the perimeter of the square to its side length. Then find the perimeter of a square with a 6-inch side.

Part A Write an equation.

The perimeter of a square with 10-inch sides is 40 inches. Write a ratio that compares the perimeter to the side length. Find the constant of proportionality. Find an equivalent ratio with a denominator of 1.

$$\frac{\text{perimeter}}{\text{side length}} = \frac{40}{10} = \frac{4}{1}$$

So, the constant of proportionality is 4.

The perimeter is 4 times the length of a side, so the equation is written $y = 4x$.

Part B Use the equation to find the perimeter of a square with a side length of 6 inches.

$$y = 4x$$

Write the equation.

$$y = 4(6)$$

Replace x with 6.

$$y = 24$$

Multiply.

So, the perimeter of a square with sides of 6 inches is _____ inches.

Check

Jack is in charge of the 120,000-gallon community swimming pool. Each spring, he drains the pool in order to clean it. When finished, he refills the pool with fresh water. Jack can fill the pool with 500 gallons of water in 5 minutes.

Part A

Write an equation that represents the relationship between the total number of gallons y and the number of minutes spent filling the pool x .

$$y = kx$$

$$y = 100x$$

Part B

How long will it take to become completely filled?

$$\frac{120,000}{100} = 1200$$

Go Online You can complete an Extra Example online.

Check

Emily can ride her bike 15 kilometers in 45 minutes. How many more kilometers can she bike in two hours than in 45 minutes? Assume the relationship is proportional and she always bikes at the same rate.



$$\begin{array}{rcl} & \text{km} & \text{min} \\ 15 & & 45 \\ & \times & 120 \end{array}$$

$$15 \times 120 = 40 \text{ km}$$

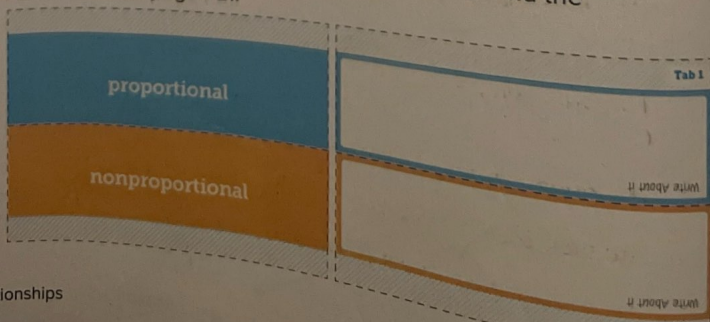
So, the distance in 2 hours
= 40 km

How many more km

$$40 - 15 = 25 \text{ km}$$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Practice

1. Liv earns \$9.50 for every two bracelets she sells. The equation $y = 4.75x$, where x represents the number of bracelets and y represents the total cost in dollars earned, represents this situation. What is the constant of proportionality? What does the constant of proportionality represent in the context of the problem? (Example 1)

$k = 4.75$
constant of proportionality
represent that liv earn
4.75 every 1 bracelet

3. Lincoln bought 3 bottles of an energy drink for \$4.50. Write an equation relating the total cost y to the number of energy drinks bought x . (Example 2)

$$y = kx$$

$$k = \frac{y}{x} = \frac{4.5}{3}$$

$$k = 1.5$$

$$y = 1.5x$$

5. Marley used 7 cups of water to make 4 loaves of French bread. What equation can be used to model the total cups of water needed y for making x loaves of French bread? How many cups of water do you need for 6 loaves of French bread? (Example 3)

$$y = kx$$

$$y = \text{water}$$

$$x = \text{loaves}$$

$$k = \frac{7}{4}$$

$$y = 1.75x$$

$$y = 1.75(6)$$

$$y = 10.5$$

Test Practice

7. **Multiselect** The table shows the cost of 4 movie tickets at two theaters. Select the statements that are true about the situation.

☒ The equation $y = 7.75x$ models the cost for tickets at Star Cinema.

☒ The equation $y = 30x$ models the cost for tickets at Movies Galore.

☒ The total cost of 9 tickets at Star Cinema would cost \$69.75.

☒ The total cost for 1 ticket at Movies Galore is \$30.

Theater	Cost (\$)
Movies Galore	30
Star Cinema	31

$$k = \frac{30}{4} = 7.5$$

$$k = \frac{31}{4} = 7.75$$

$$69.75 \div 9 = 7.75$$

2. John ran 3 miles in 25.5 minutes. The equation $y = 8.5x$, where x represents the number of miles and y represents the total time in minutes, represents this situation. What is the constant of proportionality? What does the constant of proportionality represent in the context of the problem? (Example 1)

$$k = 8.5$$

it represent that John
ran 8.5 in 1 min

4. The total cost of renting a cotton candy machine for 4 hours is \$72. What equation can be used to model the total cost y for renting the cotton candy machine x hours? (Example 2)

$$y = kx$$

$$x = \$$$

$$y = h$$

$$k = \frac{\$}{h} = \frac{72}{4} = 18$$

$$y = 18x$$

6. Mrs. Henderson used 6 $\frac{3}{4}$ yards of fabric to make 3 elf costumes. What equation can be used to model the total number of yards of fabric y for x costumes? How many yards of fabric do you need for 7 elf costumes? (Example 3)

$$y = kx$$

$$k = \frac{\text{yard}}{\text{const}}$$

$$y = \frac{27}{4} \div 3$$

$$\frac{27}{4} \times \frac{1}{3}$$

$$\frac{15}{4}$$

$$y = 2.25x$$

$$y = 2.25(7)$$

Apply

8. Roman can type 3 pages in 60 minutes. How many more pages can Roman type in 90 minutes than in 60 minutes? Assume the relationship is proportional and he types at a constant rate.

$$\frac{3 \text{ pages}}{60 \text{ minutes}} = \frac{x \text{ pages}}{90 \text{ minutes}}$$

$$4.5 - 3 = 1.5$$

$$\frac{96 \times 1}{6628} = 4.5 \text{ words}$$

9. On average, Asia has 20 free throws. Assuming the relationship is proportional, how many more free throws is she likely to make if she shoots 150 free throws?

$$\frac{14}{20} \times 150 = 105$$

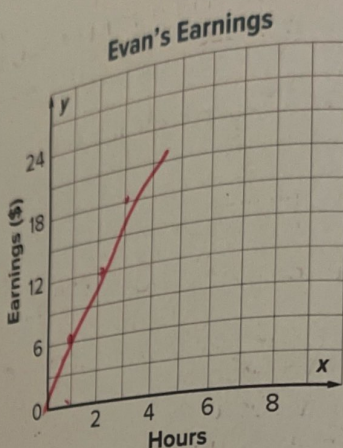
$$105 - 14 = 91 \text{ Free Throws}$$

10. Evan earned \$26 for 4 hours of babysitting. What equation can be used to model his total earnings y for babysitting x hours? Then graph the equation on the coordinate plane. What is the unit rate? How is that represented on the graph?

$$\frac{26}{4} = 6.5$$

$$y = 6.5x$$

6	5
0	1
2	12
3	18



11. **MP Persevere with Problems** The Diaz family spent \$38.25 on 3 large pizzas. What is the cost of one large pizza? Assume the situation is proportional. Explain how you solved.

$$\frac{\$38.25}{3} = \$12.75$$

12. **MP Use a Counterexample** Determine whether the statement is true or false. If false, give a counterexample.

The constant of proportionality in an equation can never be 0.

$y = 0 \rightarrow$ is Not proportional
True

$y = kx$

$k = 0$

$y = kx = 0$

13. **MP Justify Conclusions** A recipe for homemade modeling clay includes $\frac{1}{3}$ cup of salt for every cup of water. If there are 6 cups of salt, how many gallons of water are needed? Identify the constant of proportionality. Explain your reasoning.

$$6 \div \frac{1}{3} = 18 \text{ cups of water}$$

$$\frac{\text{salt}}{\text{water}} = \frac{\frac{1}{3}}{1}$$

Cup $\rightarrow$ gallons

$1 \rightarrow 0.0625$

$18 \rightarrow ?$