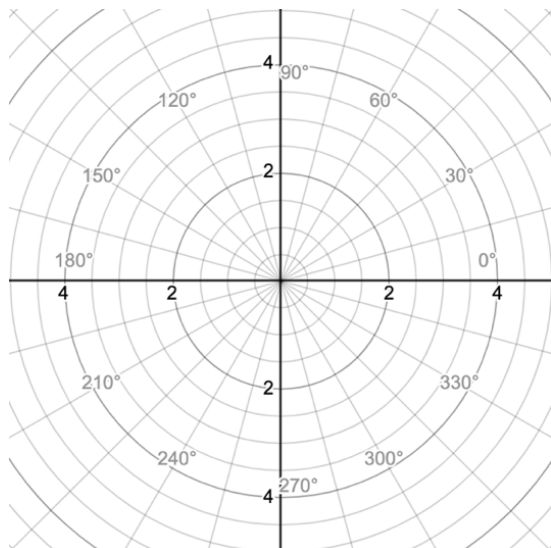


## ❖ Topic 1 Polar Coordinates and Complex Numbers

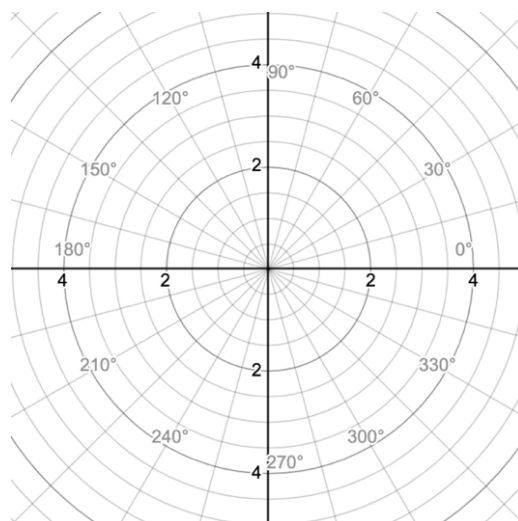
### Subtopic 1.1 Polar Coordinates

1. Graph each point on a polar grid.

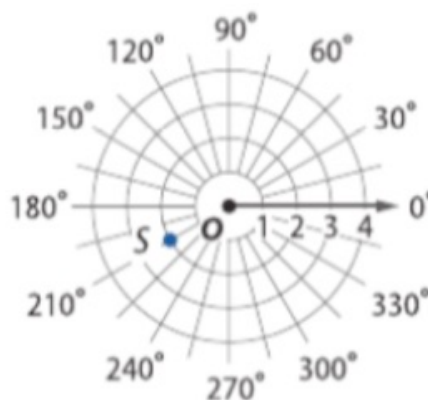
$$A\left(3, \frac{5\pi}{6}\right)$$



$$A(-2, -240^\circ)$$



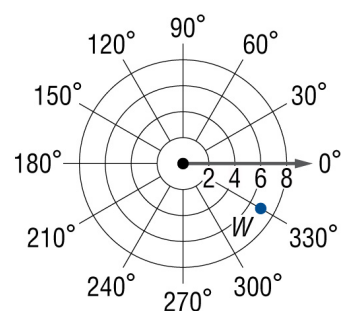
2. Find four different pairs of polar coordinates that name point S if  $-360^\circ < \theta < 360^\circ$



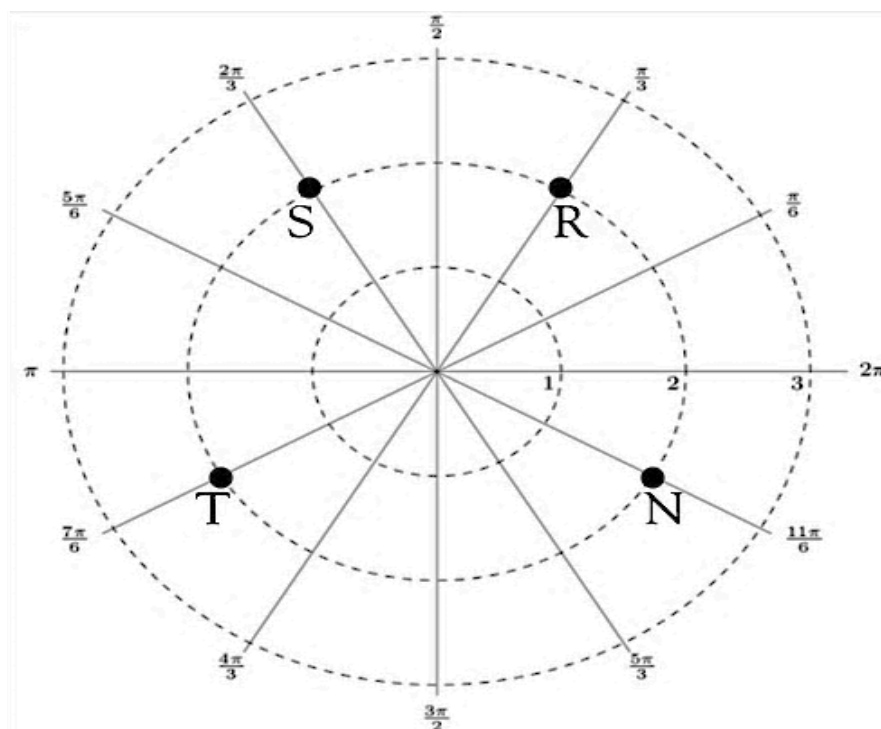
3. Which of the following Does Not represent another pair of polar coordinates for point A  $(3, 120^\circ)$ ?
- A.  $(3, -300^\circ)$
  - B.  $(-3, -60^\circ)$
  - C.  $(3, -240^\circ)$
  - D.  $(3, 480^\circ)$

4. Find four different pairs of polar coordinates that name point  $W$  if  $-360^\circ \leq \theta \leq 360^\circ$ .

- A.  $(7, -30^\circ), (7, 150^\circ), (7, 210^\circ), (7, 330^\circ)$
- B.  $(5, -30^\circ), (5, 330^\circ), (-5, 150^\circ), (-5, -210^\circ)$
- C.  $(7, -30^\circ), (7, 330^\circ), (-7, 150^\circ), (-7, -210^\circ)$
- D.  $(7, -150^\circ), (7, 330^\circ), (-7, 30^\circ), (-7, 210^\circ)$



5. Given the below grid. Answer questions (i-iii).

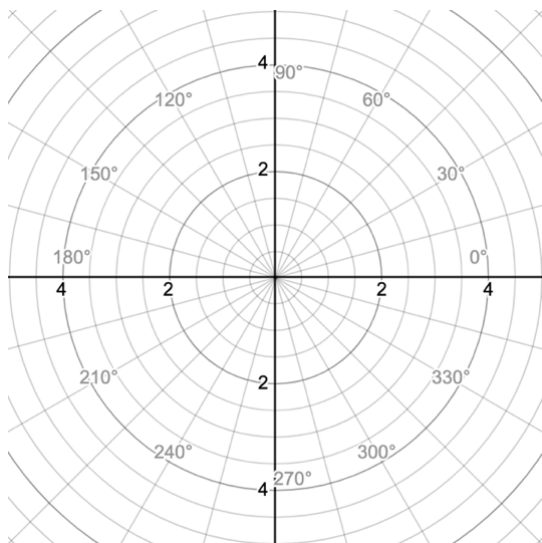
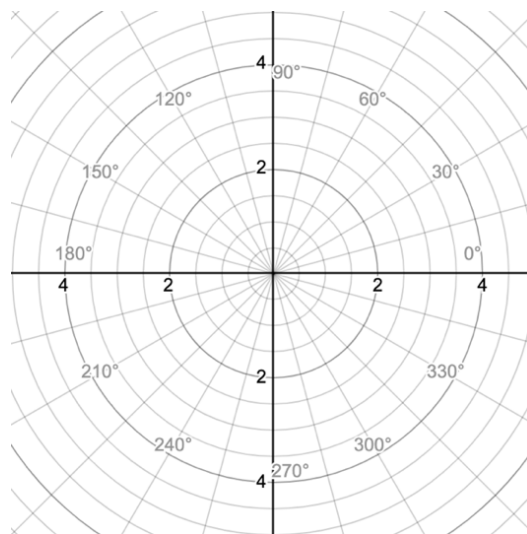


- i. The point with the polar coordinates  $(-2, 240^\circ)$  on the given polar grid, is \_\_\_\_.
- ii. The point with the polar coordinates  $(-2, 300^\circ)$  on the given polar grid, is \_\_\_\_.
- iii. The point with the polar coordinates  $(-2, 30^\circ)$  on the given polar grid, is \_\_\_\_.

6. Graph each polar equation.

$$r = 2.5$$

$$\theta = \frac{5\pi}{6}$$



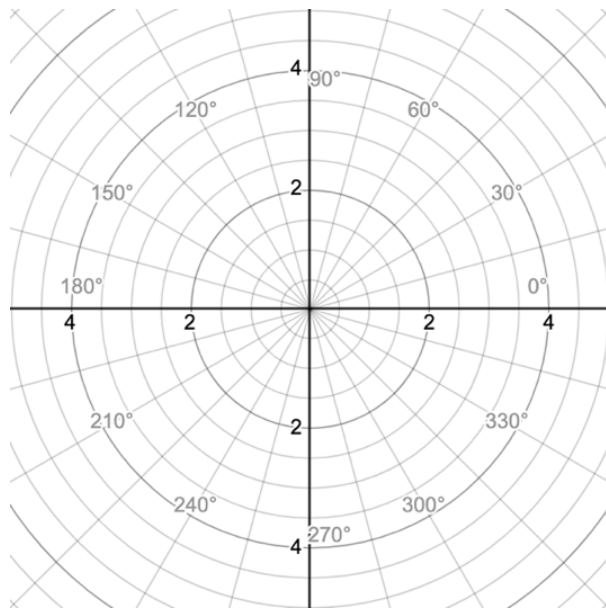
7. Find the distance  $P_1P_2$  between each pair of points to the nearest tenth degree.

$$P_1(3, 40^\circ), P_2(5, 130^\circ)$$

8. A naval radar is tracking two aircraft. The coordinates of the two carriers are  $(8, 150^\circ)$  and  $(3, 65^\circ)$  where  $r$  is measured in kilometers.  
What is the distance between the two aircraft carriers.

9. An air traffic controller is tracking two airplanes that are flying at the same altitude. The coordinates of the planes are  $A(8, 310^\circ)$  and  $B(10, 345^\circ)$ , where the directed distance is measured in kilometers.

i. Sketch the graph of the situation.



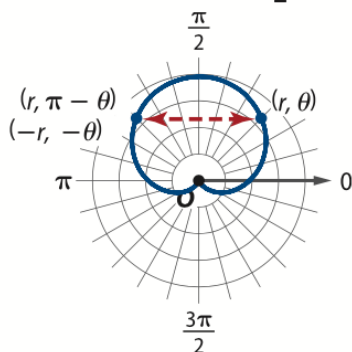
- ii. If regulations prohibit airplanes from passing within five kilometers of each other, are these airplanes in violation? Explain.

Using the polar distance formula, we have:

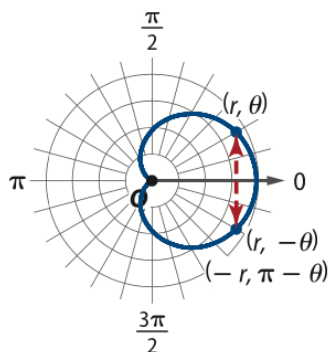
## Subtopic 1.2 Graphs of Polar Equations

### KeyConcept Symmetry of Polar Graphs

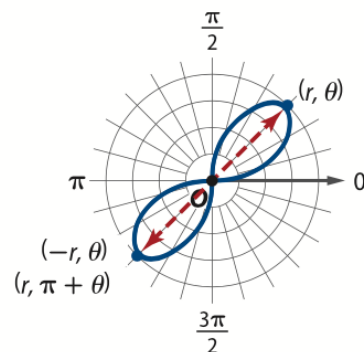
Symmetry with Respect  
to the Line  $\theta = \frac{\pi}{2}$



Symmetry with Respect  
to Polar Axis



Symmetry with Respect  
to the Pole



1. Apply the symmetry tests to prove that the graph  $r = 4\sin 3\theta$  is symmetric about  $\theta = \frac{\pi}{2}$ . (y-axis)
2. Apply the symmetry tests to prove that the graph  $r = 1 + 2\cos\theta$  is symmetric about the polar axis (x-axis).

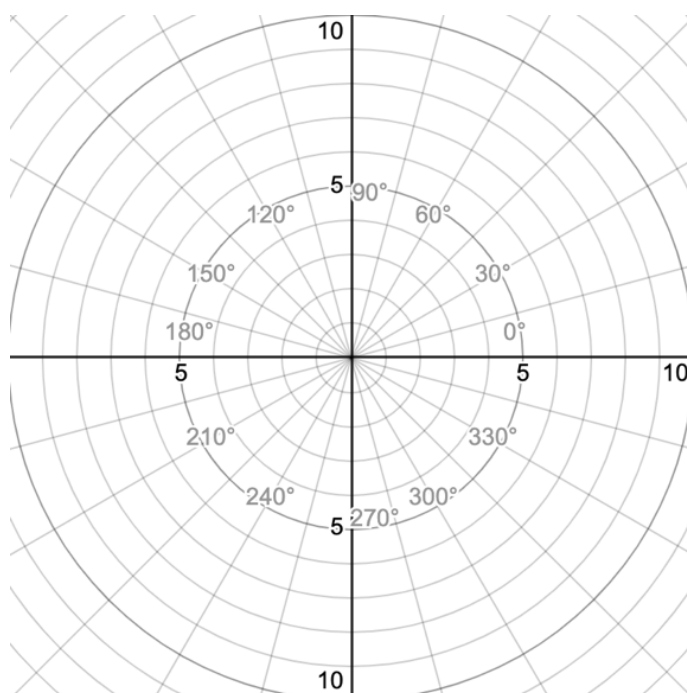
3. Apply the symmetry tests to prove that the graph  $r^2 = 15 \sin 2\theta$  is symmetric about the pole (the origin).

4. Given the polar curve  $r = -5 \cos 2\theta$ . (Answer i-ii).

i. Fill in the table below

|          |     |     |     |     |     |     |     |     |     |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\theta$ | 0   | 30  | 45  | 60  | 90  | 120 | 135 | 150 | 180 |
| $r$      |     |     |     |     |     |     |     |     |     |
| $\theta$ | 210 | 225 | 240 | 270 | 300 | 315 | 330 |     |     |
| $r$      |     |     |     |     |     |     |     |     |     |

ii. Sketch the graph of the function.

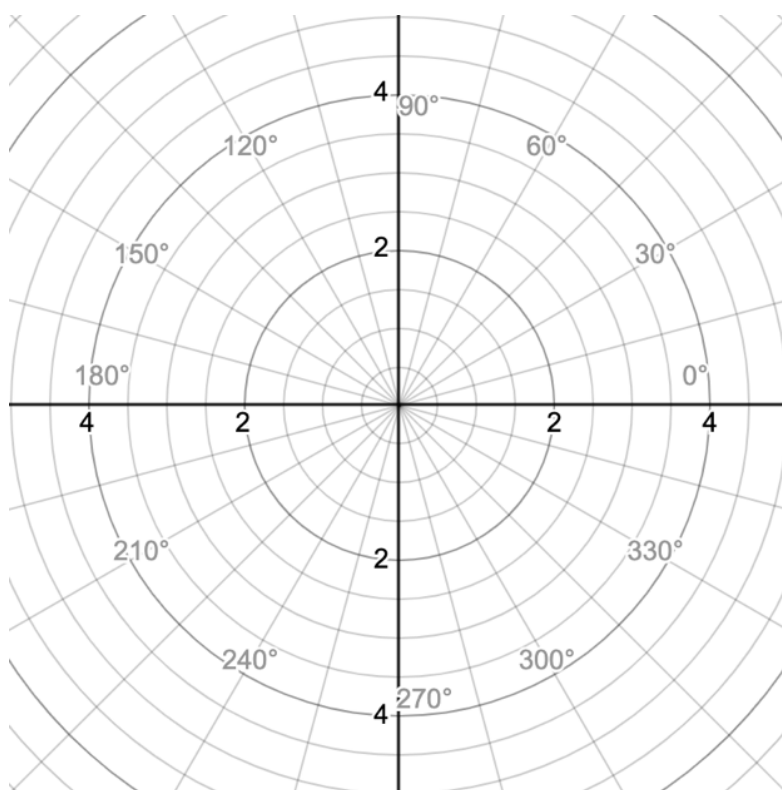


5. Given the polar curve  $r = 3 \sin 2\theta$ . Answer questions (i-ii).

i. Fill in the table below

|          |     |     |     |     |     |     |     |     |     |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\theta$ | 0   | 30  | 45  | 60  | 90  | 120 | 135 | 150 | 180 |
| $r$      |     |     |     |     |     |     |     |     |     |
| $\theta$ | 210 | 225 | 240 | 270 | 300 | 315 | 330 |     |     |
| $r$      |     |     |     |     |     |     |     |     |     |

ii. Sketch the graph of the function.



### **Subtopic 1.3 Polar and Rectangular Forms of Equations**

1. Find the **rectangular coordinates** of each point with the given polar coordinates.

i.  $P(-2, \pi)$

ii.  $Q(3, \frac{5\pi}{6})$

iii.  $S(2, 270^\circ)$

2. Which of the following represents the rectangular coordinates of the point  $P(6, \frac{\pi}{6})$ ?
- A.  $(\sqrt{3}, 3)$
  - B.  $(3\sqrt{3}, 3)$
  - C.  $(6, \pi)$
  - D.  $(\sqrt{3}, 6)$
3. The rectangular form of the polar equation  $r = 4\cos\theta$  is \_\_\_\_\_.
- A.  $x^2 + (y - 2)^2 = 4$
  - B.  $(x + 2)^2 + y^2 = 4$
  - C.  $x^2 + (y + 2)^2 = 4$
  - D.  $(x - 2)^2 + y^2 = 4$
4. Write the rectangular equation  $(x - 2)^2 + (y - 8)^2 = 68$  in its polar form  $r = \underline{\hspace{1cm}}$ , for  $r \neq 0$ .
- A.  $r = -6 \cos \theta + 4 \sin \theta$
  - B.  $r = 4 \cos \theta + 16 \sin \theta$
  - C.  $r = 12 \cos \theta + 16 \sin \theta$
  - D.  $r = 16 \cos \theta - 4 \sin \theta$
5. Given the rectangular coordinates for point  $S(3, -3\sqrt{3})$ , find the two pairs of polar coordinates for point S.

6. Convert the **polar equation to rectangular form** and identify the graph. Simplify your answer.

i.  $r = 3\sec\theta$

ii.  $r = -2\csc\theta$

iii.  $r = -2\sin\theta$

iv.  $r = -4\cos\theta$

v.  $r \csc \theta = 1$

vi.  $r = 2 \sin \theta - 4 \cos \theta$

7. Convert each **rectangular equation to polar form**. Find an expression of  $r$ .

i.  $x = 2$

ii.  $2x - 3y = 5$

iii.  $(x - 3)^2 + y^2 = 9$

iv.  $(x - 1)^2 + (y + 4)^2 = 17$

v.  $y = -x^2$

### Subtopic 1.5 Complex and De Moivre's Theorem

1. For each of the complex numbers given below, graph and find the absolute value. Also, express it in polar form.

i.  $z = 1 - \sqrt{3}i$

ii.  $z = -3 - 4i$

2. Find  $2(\cos \frac{2\pi}{9} + i \sin \frac{2\pi}{9}) \cdot 5(\cos \frac{\pi}{9} + i \sin \frac{\pi}{9})$  in polar form. Then express the product in rectangular form.

3. Find  $\frac{20(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})}{4(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})}$  in polar form. Then express the product in rectangular form.

4. Find the product of  $z_1$  and  $z_2$ . Express in rectangular form.

i.  $z_1 = 25\sqrt{2} (\cos\frac{-\pi}{4} + i \sin\frac{-\pi}{4})$   $z_2 = 14 (\cos\frac{\pi}{3} + i \sin\frac{\pi}{3})$

ii.  $z_1 = 7 (\cos 25^\circ + i \sin 25^\circ)$   $z_2 = 2 (\cos 130^\circ + i \sin 130^\circ)$

5. Use an algebraic method to express the quotient of  $z_1$  and  $z_2$

$$z_1 = 2\sqrt{2} (\cos 135^\circ + i \sin 135^\circ) \quad z_2 = 6 (\cos 300^\circ + i \sin 300^\circ)$$

6. Find  $(1 + i\sqrt{3})^3$

7. Find  $(\frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})^8$

8. Find  $(4 + 4\sqrt{3}i)^6$  and express it in a rectangular form.

9. Express the complex number  $2 - 2i$  in polar form.

- A.  $4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$
- B.  $2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$
- C.  $2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$
- D.  $3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

10. Find the product  $-6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \cdot 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ .

- A.  $-12\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$
- B.  $-3\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
- C.  $-12\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
- D.  $-3\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

11. Find the Quotient  $-6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ .

- A.  $-12\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$
- B.  $-3\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
- C.  $-12\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
- D.  $-3\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

12. Find the below power and express it in rectangular form.

$$\left[4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]^4$$

- A. 256
- B. 64
- C.  $256i$
- D.  $256 + 16i$

13. Find the fourth roots of  $z = 5(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$

14. Find the cube roots of  $z = -1$

15. Answer questions (i-iv).

- i. Write the complex number  $z_1 = 4 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$  in rectangular form.
- ii. Given  $z_1 = 4 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$  and  $z_2 = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$ , find the product  $z_1 \cdot z_2$ .
- iii. Use De Moivre's Theorem to find the **fifth power** of the complex number  $z_2 = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$ .
- iv. Find **any one** of the cube roots of the complex number  $z_1 = 4 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$ .