

❖ Topic 1 – Polynomials

Subtopic 1.1 Adding and Subtracting Polynomials

1. Select all that are polynomials.

- ✓ A. $49x^2 - 64$
- B. $-7xy + \frac{2}{y}$
- ✓ C. $45y^2 + y - y^3 + 8 + x^3$
- ✓ D. -16
- E. $3x^{-2} + 6x^2$
- F. $49\sqrt{x} - 64$

2. Select all of the statements that are true.

- A. 25 is a monomial of degree 1.
- ✓ B. $5x^2 + 3\sqrt{x} + 1$ is not a polynomial.
- C. $8y^5 - 2y^3 + 27y^2$ is a trinomial of degree 10.
- ✓ D. The leading coefficient of $8y + 6y^3 + 2y^5$ is 2.
- E. $3xy^5 - 7x^2$ is a polynomial of degree 5.

3. Find $(3 - 4x - 9x^2) - (-8x^2 + 7)$.

- A. $x^2 - 4x - 4$
- ✓ B. $-x^2 - 4x - 4$
- C. $-17x^2 - 4x + 10$
- D. $-11x^2 + 3x - 9x^2$

4. Answer the following questions (i-ii).

i. Write the following polynomial in standard form. Then identify the leading coefficient.

$$4b^5 + 2b - 17b^6 - 4b^2$$

Standard form: $-17b^6 + 4b^5 - 4b^2 + 2b$.

Leading Coefficient: -17 .

ii. Find the difference: $(5d^2 - 4cd + 3c^2) - (d^2 - 9cd - c^2)$

$$5d^2 - 4cd + 3c^2 - d^2 + 9cd + c^2$$

$$4d^2 + 5cd + 4c^2$$

5. Determine whether the given expression is a polynomial. If it is a polynomial, find the degree, and determine whether it is monomial, binomial, trinomial, or a polynomial with more than three terms

$$8ab - 3g$$

It is a Polynomial. Degree: 2; binomial

$$-101.5$$

It is a Polynomial. Degree: 0; monomial

$$2x^{-2} + 12xy$$

It is not a Polynomial.

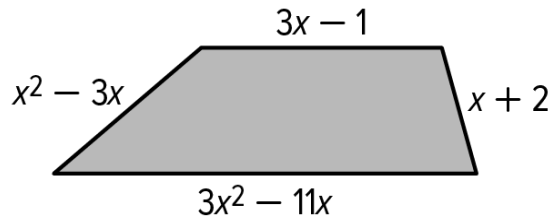
$$2x^3 - 16x + 5x - 27$$

It is a Polynomial. Degree: 3; trinomial (*The polynomial can be written as $2x^3 - 11x - 27$*)

$$9x^4 - 8x^2 + 5x - 27$$

It is a Polynomial. Degree: 4; more than three terms

6. What is the perimeter of this figure?



Perimeter = 4 x^2 + -10 x + 1

(Only final answer is given. Show your work)

7. The volume of Box U is $(6x^2 + 8 - x + 9x^3)$. The volume of Box V is $(3x^3 + 7x + 5)$. Write an expression in standard form to represent the difference of the between the volume of Box U and the volume of Box V.

$6x^3 + 6x^2 - 8x + 3$

(Only final answer is given. Show your work)

Subtopic 1.2 Multiplying Monomials and Polynomials

1. Select the statements that are sometimes true.

- A. The product of two monomials is a binomial.
- ✓ B. The product of two binomials is a trinomial.
- ✓ C. The product of a binomial and a trinomial is a polynomial with four terms.
- ✓ E. The product of two binomials is a binomial.

2. Find the product of $(4h + 5)$ and $(h - 6)$.

- A. $5h - 1$
- ✓ B. $4h^2 - 19h - 30$
- C. $4h^2 - 24h - 30$
- D. $4h^2 - 30$

3. Find $(7x + 3)^2$

- A. $21x^2 + 42x + 9$
- ✓ B. $49x^2 + 42x + 9$
- C. $49x^2 + 9$
- D. $28x^2 + 42x + 18$

4. Find the product:

$$(2c^3 - 3c + 1)(4c^2 - 2c + 5)$$

$$8c^5 - 4c^4 + 10c^3 - 12c^3 + 6c^2 - 15c + 4c^2 - 2c + 5$$

$$8c^5 - 4c^4 - 2c^3 + 10c^2 - 17c + 5$$

5. Solve $2h(h - 5) - 3h(h + 4) = h(6 - h) - 56$. (Only final answer is given. Show your work).

$h = \underline{\textcolor{red}{2}}\underline{\hspace{0.5cm}}$

6. Solve the below equation, for q :

$$-2q(4q - 9) - q(-8q + 15) = 3(4q + 6)$$

☒ A. -2

B. 2

C. $\frac{2}{5}$

D. $\frac{4}{3}$

7. Solve the equation $x(x + 2) + 4x = x^2 + 6$ for x .

A. 0

☒ B. 1

C. 2

D. 3

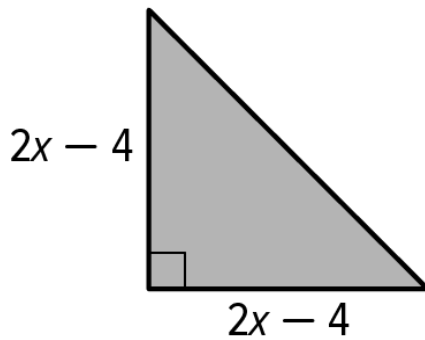
8. Simplify the below expression:

$$(x^3 - 2x)(x^4 + 1)$$

$$\textcolor{red}{x^7 + x^3 - 2x^5 - 2x}$$

$$\textcolor{red}{x^7 - 2x^5 + x^3 - 2x}$$

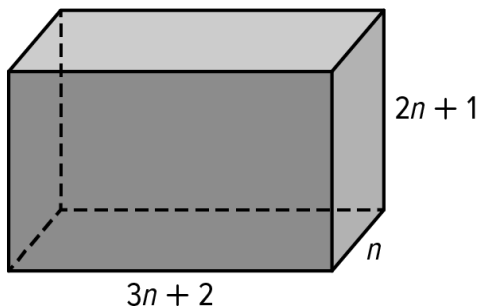
9. The right triangle shown has two sides that are each $2x - 4$ inches long.



If the area of a triangle is $A = \frac{1}{2}bh$, where b is the base and h is the height of the triangle, what is the area of the triangle shown above? (Write in simplest form).

- A. $A = 4x^2 - 16x + 16$
- ✓ B. $A = 2x^2 - 8x + 8$
- C. $A = 2x^2 + 8x - 8$
- D. $A = 2x^2 - 4x + 8$

10. A cardboard box company designs all of its boxes so that they have a depth of n , a height of $2n + 1$, and a width of $3n + 2$.

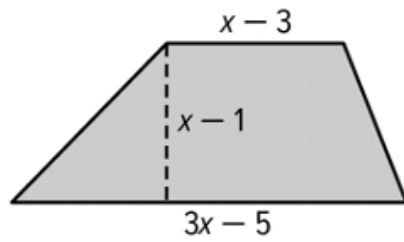


Write an expression in standard form for the volume V of one of these boxes.

(Only final answer is given. Show your work).

____ $6n^3 + 7n^2 + 2n$ ____

11. The area of a trapezoid is one-half the height times the sum of the bases, $A = \frac{1}{2}h(b_1 + b_2)$.



The area of the trapezoid at the right is _____.

$$A = \frac{1}{2}(x - 1)((x - 3) + (3x - 5))$$

$$A = \frac{1}{2}(x - 1)(4x - 8)$$

$$A = \frac{1}{2}(4x^2 - 8x - 4x + 8)$$

$$A = \frac{1}{2}(4x^2 - 12x + 8)$$

$$A = 2x^2 - 6x + 4$$

12. A horse farm is developing a new indoor arena in a rectangular building. They want the width of the building to be $4y$ feet and the length to be $2y + 9$ feet. Write an expression in simplest form to represent the area A of the building in terms of y .

$$A = 4y(2y + 9) = 8y^2 + 36y$$

13. Solve the equation $5b^2 - 5b + 7(b + 4) = 5(b^2 + 4)$.

$$5b^2 - 5b + 7b + 28 = 5b^2 + 20$$

$$-5b + 7b = 20 - 28$$

$$2b = -8$$

$$b = -4$$

14. Find each product.

i. $(2a - 3b)^2$

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2 && \text{Square of a difference} \\ (2a - 3b)^2 &= (2a)^2 - 2(2a)(3b) + (3b)^2 && a = 2a, b = 3b \\ &= 4a^2 - 12ab + 9b^2 && \text{Simplify.}\end{aligned}$$

ii. $(g + 5h)^2$

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 && \text{Square of a sum} \\ (g + 5h)^2 &= (g)^2 + 2(g)(5h) + (5h)^2 && a = g, b = 5h \\ &= g^2 + 10gh + 25h^2 && \text{Simplify.}\end{aligned}$$

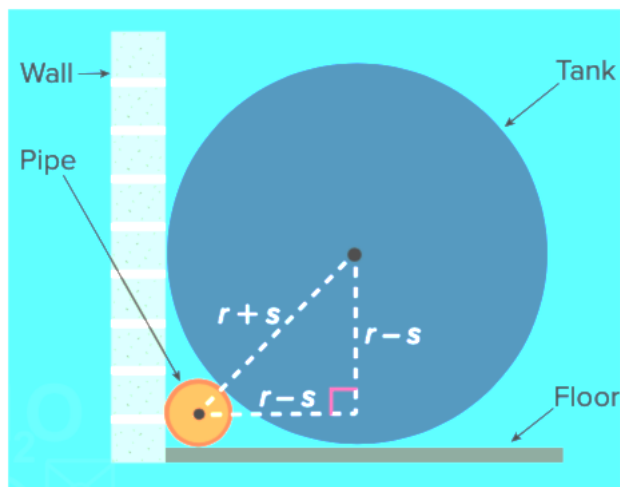
iii. $(6y - 13)(6y + 13)$

$$\begin{aligned}(a + b)(a - b) &= a^2 - b^2 && \text{Product of a sum and a difference.} \\ (6y - 13)(6y + 13) &= (6y)^2 - (13)^2 && a = 6y, b = 13 \\ &= 36y^2 - 169 && \text{Simplify.}\end{aligned}$$

iv. $(2m + 3)(2m - 3)(m + 4)$

$$\begin{aligned}(a + b)(a - b) &= a^2 - b^2 && \text{Product of a sum and a difference.} \\ (2m + 3)(2m - 3)(m + 4) &= [(2m)^2 - (3)^2](m + 4) && a = 2m, b = 3 \\ &= (4m^2 - 9)(m + 4) && \text{Simplify.} \\ &= 4m^2(m) + 4m^2(4) - 9(m) - 9(4) && \text{FOIL Method} \\ &= 4m^3 + 16m^2 - 9m - 36 && \text{Multiply.}\end{aligned}$$

15. A cylindrical tank is placed along a wall. A cylindrical PVC pipe will be hidden in the corner behind the tank. See the side-view diagram shown. The radius of the tank is r inches, and the radius of the PVC pipe is s inches.



- i. Use the Pythagorean Theorem to write an equation for the relationship between the two radii. Simplify your equation so that there is a zero on one side of the equal sign.

$$\begin{aligned}(r - s)^2 + (r - s)^2 &= (r + s)^2 \\ [(r)^2 - 2(r)(s) + (s)^2] + [(r)^2 - 2(r)(s) + (s)^2] &= (r)^2 + 2(r)(s) + (s)^2 \\ r^2 - 2rs + s^2 + r^2 - 2rs + s^2 &= r^2 + 2rs + s^2 \\ 2r^2 - 4rs + 2s^2 &= r^2 + 2rs + s^2 \\ r^2 - 6rs + s^2 &= 0\end{aligned}$$

- ii. Write a polynomial equation you could solve to find the radius s of the PVC pipe if the radius of the tank is 20 inches.

$$\begin{aligned}r^2 - 6rs + s^2 &= 0 \\ r^2 - 6r(20) + (20)^2 &= 0 \\ r^2 - 120r + 400 &= 0\end{aligned}$$

Subtopic 1.3 Using the Distributive Property

1. What is the greatest common factor (GCF) of $40xy^2 - 60x^3y^4 + 80x^4y^2$?

_____ $20xy^2$ _____

2. If the area of a given parallelogram is represented by the expression $3x^2 + 15x - 2xy - 10y$, what is the factored form of this expression?

(Only final answer is given. Show your work).

_____ $(3x - 2y)(x + 5)$ _____

3. Factor $6xy^3 + 18x^2y^4$.

- ✓ A. $6xy^3(1 + 3xy)$
B. $6xy^3(y + 3xy)$
C. $6xy^2(y + 3xy)$
D. Prime

4. Factor $6xy - 8x - 15y + 20$

- A. $(2x + 5)(3y + 4)$
B. $(2x + 5)(3y - 4)$
C. $(2x - 5)(3y + 4)$
✓ D. $(2x - 5)(3y - 4)$

5. An expression for the area of a rectangle is $xy - 2x + 3y - 6$. Express this area in factored form.

- ✓ A. $(x + 3)(y - 2)$
B. $(xy - 2x)(3y - 6)$
C. $x(y - 2) + 3(y - 2)$
D. $(x - 2)(y + 3)$

6. Factor $3x^3 + 2x^2y + 3xy^2 + 2y^3$

- A. $x^2(3x + 2y) + y^2(3x + 2y)$
- ✓ B. $(x^2 + y^2)(3x + 2y)$
- C. $xy(3x^2 + 2x + 3y + 2y^2)$
- D. $(3x + 2y)(x + y)(x - y)$

7. Factor each of the polynomials below:

i. $10x^2 + 100x^3z$

$$10x^2(1 + 10xz)$$

ii. $2x^3z - 6x^3y + 8x^2z$

$$= 2x^2(xz - 3xy + 4z)$$

iii. $10x^2y^{11}z^3 - x^5y^9z + 2x^2yz^3 + x^2yz$

$$= x^2yz(10y^{10}z^2 - x^3y^8 + 2z^2 + 1)$$

iv. $2uv + 6u + 5v + 15$

$$\begin{aligned} &= 2u(v + 3) + 5(v + 3) \\ &= (2u + 5)(v + 3) \end{aligned}$$

Subtopic 1.4 Factoring Quadratic Trinomials

1. Factor $x^2 + 5x + 6$.

- ✓ A. $(x + 2)(x + 3)$
- B. $(x + 6)(x + 1)$
- C. $(x - 2)(x - 3)$
- D. $(x + 2)(x + 4)$

2. Factor $x^2 - 7x + 6$.

- ✓ A. $(x - 6)(x - 1)$
- B. $(x - 6)(x + 1)$
- C. $(x + 6)(x - 1)$
- D. Prime

3. Factor $x^2 + x - 12$.

- A. $(x - 3)(x - 4)$
- ✓ B. $(x - 3)(x + 4)$
- C. $(x + 3)(x - 4)$
- D. Prime

4. Factor $4x^2 - 35x + 49$.

- ✓ A. $(x - 7)(4x - 7)$
- B. $(x - 7)(4x + 7)$
- C. $(x + 7)(4x - 7)$
- D. Prime

5. Factor $3x^2 - 2x - 5$.

- A. $(3x - 5)(x - 1)$
- B. $(3x + 5)(x - 1)$
- ✓ C. $(3x - 5)(x + 1)$
- D. Prime

6. Factor the below expressions:

$$x^2 - 5x - 36$$

$$(x - 9)(x + 4)$$

$$4x^2 - 17x + 4$$

$$4x^2 - 16x - x + 4$$

$$(4x^2 - 16x) - (x - 4)$$

$$4x(x - 4) - (x - 4)$$

$$(4x - 1)(x - 4)$$

$$2x^2 + 11x + 5$$

$$2x^2 + 10x + x + 5$$

$$(2x^2 + 10x) + (x + 5)$$

$$2x(x + 5) + (x + 5)$$

$$(2x + 1)(x + 5)$$

7. Solve the equation $x^2 + 3x - 4 = 0$

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

$$x + 4 = 0 \text{ or } x - 1 = 0$$

$$x = -4 \text{ or } x = 1$$

8. The height of a projectile in feet is given by $-16t^2 + vt + h_0$, where t is the time in seconds, v is the initial upward velocity in feet per second, and h_0 is the initial height in feet. A T-shirt is propelled from 32 feet above ground level into the air at an initial velocity of 16 feet per second.

- i. Write an expression to represent how much time the T-shirt is in the air.

Substitute 16 for v and 32 for h_0 into $-16t^2 + vt + h_0$ to get $-16t^2 + 16t - 32$.

- ii. Factor the expression that represents the amount of time the T-shirt is in the air.

$$\begin{aligned} -16t^2 + 16t - 32 &= -16(t^2 - t - 2) \\ &= -16(t - 2)(t + 1) \end{aligned}$$

Subtopic 1.5 Factoring Special Products

1. Factor $x^2 - 8x + 16$

- A. $(x + 4)^2$
- B. $(x + 2)^2$
- C. $(x - 2)^2$
- ✓ D. $(x - 4)^2$

2. Factor $4x^2 - 121$.

- A. $(x - 11)(x + 11)$
- B. $(2x - 11)(2x - 11)$
- ✓ C. $(2x - 11)(2x + 11)$
- D. Prime

3. Select all polynomials that can be factored using the differences of squares.

- ✓ A. $-4y^2 + k^2$
- ✓ B. $49x^2 - 64y^2$
- C. $-4y^2 - k^2$
- ✓ D. $m^2 - 10,000$
- ✓ E. $121 - 16x^2y^2$

4. Factorize the below expression:

$$9x^2 - 6x + 1$$

$$=(3x)^2 - 2(3x) + 1$$

$$= (3x - 1)^2$$

5. Explain how you could use mental math and the differences of squares to find the product of 28 and 32.

Sample answer: Because $a^2 - b^2$, a difference of squares, is equivalent to $(a - b)(a + b)$, think of 28 as $(30 - 2)$ and 32 as $(30 + 2)$. So, $28(32) = (30 - 2)(30 + 2) = 900 - 4$ (a difference of squares), or 896.

6. Factor $5x^3 - 20x$ completely.

- A. $5x(x - 4)$
- B. Prime
- C. $5x(x + 2)(x + 2)$
- ✓ D. $5x(x + 2)(x - 2)$

7. Factor $25a^2 - 36b^2$

- ✓ A. $(5a + 6b)(5a - 6b)$
- B. $(5a + 3b)(5a + 3b)$
- C. $(5a - 6b)^2$
- D. $(5a + 6b)^2$

8. Find x such that $(x + 6)^2 = x^2 - 6^2$.

$$(x + 6)^2 = x^2 - 6^2 \rightarrow x^2 + 12x + 6^2 = x^2 - 6^2$$

$$12x = -6^2 - 6^2 = -72$$

$$x = -6$$

9. Consider the polynomial $P(x) = (x - 1)^2 - (x - 5)^2$.

i. Expand and reduce $P(x)$.

$$P(x) = x^2 - 2x + 1 - (x^2 - 10x + 25)$$

$$P(x) = x^2 - 2x + 1 - x^2 + 10x - 25$$

$$P(x) = 8x - 24$$

ii. Factorize $P(x)$.

$$P(x) = 8(x - 3)$$

iii. Factorize $P(x)$ without expanding. How does your answer compare to your answer in part ii?

$$P(x) = ((x - 1) - (x - 5))((x - 1) + (x - 5))$$

$$P(x) = (4)(2x - 6) = 8(x - 3)$$

Both answers are equivalent.

iii. Solve the equation $P(x) = 8$.

$$P(x) = 8(x - 3) = 8$$

$$(x - 3) = 1$$

$$x = 4$$

10. Answer questions (i-iii)

i. Factor $x^3 - 6x^2 - 4x + 24$ completely.

$$x^2(x - 6) - 4(x - 6)$$

$$(x^2 - 4)(x - 6)$$

$$(x - 2)(x + 2)(x - 6)$$

ii. Factor $x^4 - 14x^3 + 49x^2$ completely.

Method-1:

$$= x^2(x^2 - 14x + 49)$$

$$= x^2(x - 7)(x - 7) \quad \text{OR} \quad x^2(x - 7)^2$$

Method-2:

$$= x^2(x^2 - 14x + 49)$$

$$= x^2(x^2 - 7x - 7x + 49)$$

$$= x^2(x(x - 7) - 7(x - 7))$$

$$= x^2(x - 7)(x - 7) = x^2(x - 7)^2$$

iii. Find the length and width of a rectangle with the given area: $A = x^2 + 7x - 78$.

$$A = (x - 6)(x + 13)$$

$$\text{Length} = x + 13$$

$$\text{Width} = x - 6$$