

Zu Chongzhi

Math History Minute

Various values for π have been used throughout history. Chinese mathematician **Zu Chongzhi (429–500 B.C.E.)** calculated

π as $\frac{355}{113}$, or

3.1415929... Later,

Hindu mathematician

Aryabhata (476–550

C.E.) calculated π

as $\frac{62,843}{20,000}$. In 1150,

another Hindu

mathematician

Bhāskara II (1114–

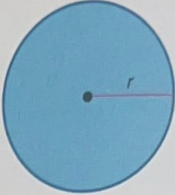
1185) obtained a value

of $\frac{3,927}{1,250}$, or 3.1416.

Learn Area of Circles

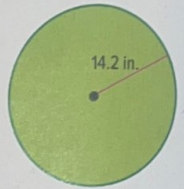
Area is the measure of the interior surface of a two-dimensional figure. As with the area of polygons, the area of a circle is expressed in square units.

The table shows the use of the formula to find the area of a circle, given the radius.

Words	Model
The area A of a circle equals the product of π and the square of the radius r .	
Symbols	
$A = \pi r^2$	

Example 1 Find the Area Given the Radius

Find the area of the circle. Use 3.14 for π . Round to the nearest hundredth if necessary.



$A = \pi r^2$

Area of a circle

$A = \pi(14.2)^2$

Replace r with 14.2.

$A = 201.64\pi$

Simplify. This is the exact area.

$A \approx 201.64(3.14)$

Replace π with 3.14.

$A \approx 633.1496$

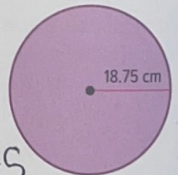
Simplify. This is the approximate area.

So, the approximate area of the circle is **633.15** square inches.

Check

Find the area of the circle. Use 3.14 for π .

Write your answer as a decimal rounded to the nearest hundredth.



$A = \pi r^2$

$A = 3.14 \times 18.75 \times 18.75$

$A \approx 1,103.91 \text{ cm}^2$

Go Online You can complete an Extra Example online.

If the Radius is doubles so, the area is quadruples because the radius is squared in area formula

Example 2 Find the Area Given the Diameter

The city of Wellington is commissioning a statue to honor their former mayor. The circular base of the statue will be 26 feet in diameter.

What is the area of the space needed to fit the base of the statue? Use 3.14 for π . Round to the nearest hundredth if necessary.

Step 1 Find the radius of the circle.

Because the diameter of the base of the statue is 26 feet, the radius of the base is $26 \div 2$ or

13 feet.

Step 2 Calculate the area of the circle.

$$A = \pi r^2 \quad \text{Area of a circle}$$

$$A = \pi (13)^2 \quad \text{Replace } r \text{ with } 13.$$

$$A = 169\pi \quad \text{Simplify. This is the exact area.}$$

$$A \approx 169(3.14) \quad \text{Replace } \pi \text{ with } 3.14.$$

$$A \approx 530.66 \quad \text{Simplify. This is the approximate area.}$$

So, the area of the space needed to fit the base of the statue is about 530.66 square feet.

Check

The circular area covered by a lawn sprinkler has a 24.25-foot diameter. What is the area of the space covered by the sprinkler? Use 3.14 for π . Round to the nearest hundredth if necessary.

Show your work here

$$A = \pi r^2$$

$$A = 3.14 \times 12.125 \times 12.125$$

$$A \approx 461.63 \text{ ft}^2$$

$$d = 24.25$$

$$r = \frac{24.25}{2}$$

$$= 12.125$$

Think About It!

What is a good estimate for the area of the base of the statue? Explain how you calculated that estimate.

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Talk About It!

How does the solution compare to your estimate?

I estimated that

the area of the base of the statue was about $13 \times 13 \times 3.14$

or 507 ft^2 . Because

$507 \approx 530.66$

my solution was reasonable

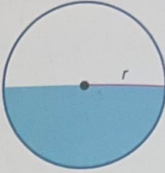
Go Online You can complete an Extra Example online.

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Mrs/AYA

Learn Area of Semicircles

A **semicircle** is half of a circle.

The table shows the use of the formula to find the area of a semicircle, given the radius.

Words	Model
The area A of a semicircle equals half the product of π and the square of the radius r .	
Symbols	
$A = \frac{1}{2}\pi r^2$	

Think About It!

What is a good estimate for the area of the space the dog has to roam?

Explain how you calculated that estimate.

I estimate that the area of the space the dog has to roam was about

$$\frac{1}{2} \times 3 \times 30 \times 30 = 1350$$

Because $1350 \approx 1413$

Reasonable

Talk About It!

How does the solution compare to your estimate?

Example 3 Find Area of Semicircles

A wireless fence transmitter at the back door of a house allows a dog to roam freely within a semicircle that has a radius of 30 feet.



What is the area of the space the dog has to roam? Use 3.14 for π . Round to the nearest hundredth if necessary.

$$A = \frac{1}{2}\pi r^2$$

Area of a semicircle

$$A = \frac{1}{2}\pi (30)^2$$

Replace r with 30.

$$A = 450\pi$$

Simplify. This is the exact area.

$$A \approx 450(3.14)$$

Replace π with 3.14.

$$A \approx 1,413$$

Simplify. This is the approximate area.

So, the dog has an approximate roaming area of 1,413 square feet.

Check

What is the area of the semicircle? Use 3.14 for π . Write your estimate as a decimal rounded to the nearest hundredth.



$$A = \frac{\pi r^2}{2}$$

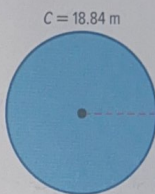
$$A = 3.14 \times \frac{7.5}{2} \times \frac{7.5}{2} \div 2$$
$$3.14 \times 3.75 \times 3.75$$
$$\underline{\hspace{2cm}}$$
$$2$$

Go Online You can complete an Extra Example online. $= 22.08 \text{ yd}^2$

Learn Use Circumference to Find Area

When you know the circumference of a circle, you can work backward to find the area of the circle.

Go Online Watch the animation to learn how to find the area of the circle, given its circumference.



Step 1 Find the radius of the circle.

$$C = 2\pi r$$

Circumference of a circle

$$18.84 \approx 2 \cdot 3.14 \cdot r$$

Replace C with 18.84 and π with 3.14.

$$18.84 \approx 6.28r$$

Simplify.

$$\frac{18.84}{6.28} \approx \frac{6.28r}{6.28}$$

Division Property of Equality

$$3 \approx r$$

Simplify.

Step 2 Find the area of the circle.

$$A = \pi r^2$$

Area of a circle

$$A \approx 3.14 \cdot 3^2$$

Replace π with 3.14 and r with 3.

$$A \approx 28.26$$

Simplify.

The area of the circle is about 28.26 m^2 .

To find Area

↓
You must have
Radius

↓
To find Radius

↓
we have to
use circumference
 $C = 2\pi r$

Think About It!

What information do you need to find the area of a circle?

Radius

Talk About It!

What does the number 32 represent when the circumference is 32π ?

32 is diameter of the circle

Example 4 Use Circumference to Find Area

The exact circumference of a circle is 32π inches.

What is the approximate area of the circle? Use 3.14 for π . Round to the nearest hundredth if necessary.

Step 1 Use the circumference formula to find the radius of the circle.

$$C = 2\pi r$$

Circumference of a circle

$$32\pi = 2\pi r$$

Replace C with 32π .

$$\frac{32\pi}{2\pi} = \frac{2\pi r}{2\pi}$$

Division Property of Equality; Divide each side by 2π .

$$16 = r$$

Simplify.

The radius of the circle is 16 inches.

Step 2 Find the area.

$$A = \pi r^2$$

Area of a circle

$$A \approx 3.14 \cdot 16^2$$

Replace π with 3.14 and r with 16.

$$A \approx 803.84$$

Simplify.

So, the approximate area of the circle is 803.84 square inches.

Check

The exact circumference of a circle is 13π feet. What is the approximate area of the circle? Use 3.14 for π . Round to the nearest hundredth.

Show your work here

$$A = \pi r^2$$

$$C = 2\pi r$$

$$\frac{13\pi}{2\pi} = \frac{2\pi r}{2\pi}$$

$$6.5 = r$$

$$A = 3.14 \times 6.5 \times 6.5 \approx 132.67 \text{ ft}^2$$

Go Online You can complete an Extra Example online.

Apply Crafting

A square scrapbook page has an area of 144 square inches. Jillian wants to cut the largest circle possible from the page to create a layered background for a new page. What is the approximate area of the paper circle? Use 3.14 for π . Round to the nearest hundredth if necessary.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.

Second Time What mathematics do you see in the problem?

Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

By drawing a circle in the square and by sure that the circumference of the circle is touching the 4 sides of the square to find the radius of the circle and the area.

3 What is your solution?

Use your strategy to solve the problem.

(1) $L^2 = 144$ $L = \sqrt{144}$
 $= 12 \text{ in}$

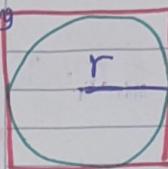
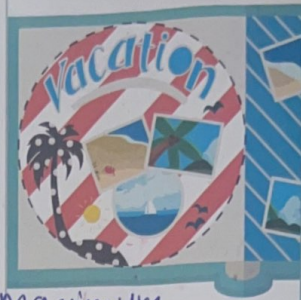
The Radius is $\frac{12}{2} = 6 \text{ in}$

Area = πr^2 $3.14 \times 6 \times 6 = 3.14 \times 36 = 113.04 \text{ in}^2$

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

My solution is reasonable because the largest circle is gotten from the largest radius and also the largest area is calculated.

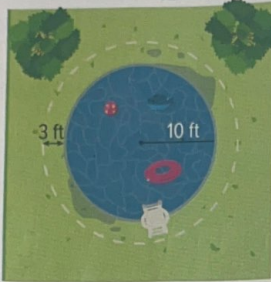


Talk About It!

How did you know that the radius of the circle is one-half the side length of the square?

Check

The Blackwells have a circular pool with a radius of 10 feet. They want to install a 3-foot wide sidewalk around the pool.



What will be the area of the sidewalk? Use 3.14 for π . Round to the nearest hundredth if necessary.

Show your work here

$$\pi r_1^2 - \pi r_2^2$$

$$\pi (r_1^2 - r_2^2)$$

$$3.14 (10+3)^2 - 10^2$$

$$3.14 \times 69 = 216.66 \text{ ft}^2$$

r of pool = 10
 r of sidewalk = 10+3 = 13

Go Online You can complete an Extra Example online.

Pause and Reflect

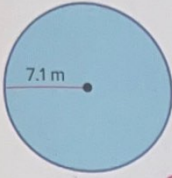
Describe when it is beneficial to use 3.14 instead of π , and when it is beneficial to use π instead of 3.14 when calculating the circumference or area of a circle.

Record your observations here

Practice

Go Online You can complete your homework online.

1. Find the area of the circle. Use 3.14 for π . Round to the nearest hundredth if necessary. (Example 1)



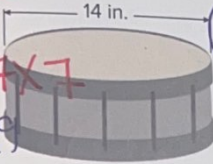
$$A = \pi r^2$$

$$3.14 \times 7.1 \times 7.1$$

$$= 158.287$$

$$\approx 159.29$$

3. What is the area of the drumhead on the drum? Use 3.14 for π . Round to the nearest hundredth if necessary. (Example 2)



$$D = 14$$

$$R = \frac{14}{2}$$

$$= 7$$

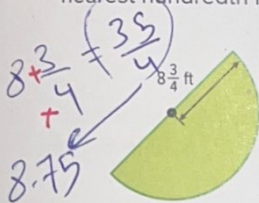
$$A = \pi r^2$$

$$A = 3.14 \times 7 \times 7$$

$$3.14 \times 49$$

$$153.86 \text{ in}^2$$

5. Mr. Ling is adding a pond in the shape of a semicircle in his backyard. What is the area of the pond? Use 3.14 for π . Round to the nearest hundredth if necessary. (Example 3)



$$A = \frac{\pi r^2}{2}$$

$$3.14 \times 8.75 \times 8.75$$

$$\frac{\quad}{2}$$

$$= 120.20 \text{ ft}^2$$

7. The exact circumference of a circle is 18π inches. What is the approximate area of the circle? Use 3.14 for π . Round to the nearest hundredth if necessary. (Example 4)

$$C = 2\pi r$$

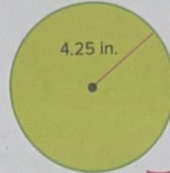
$$\frac{18\pi}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = 9$$

$$A = \pi r^2$$

$$A = 3.14 \times 9 \times 9 = 254.34$$

2. Find the area of the circle. Use 3.14 for π . Round to the nearest hundredth if necessary. (Example 1)



$$A = \pi r^2$$

$$3.14 \times 4.25 \times 4.25$$

$$= 56.706$$

$$\approx 56.72$$

4. What is the area of one side of the penny? Use 3.14 for π . Round to the nearest hundredth if necessary. (Example 2)



$$A = \pi r^2$$

$$A = 3.14 \times 9.5 \times 9.5$$

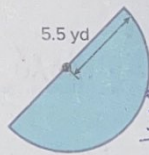
$$A = 283.39 \text{ mm}^2$$

$$D = 19$$

$$R = \frac{19}{2}$$

$$r = 9.5$$

6. Vidur needs to buy mulch for his garden. What is the area of his garden? Use 3.14 for π . Round to the nearest hundredth if necessary. (Example 3)



$$A = \frac{\pi r^2}{4}$$

$$3.14 \times 5.5 \times 5.5$$

$$\frac{\quad}{4}$$

$$= 47.49 \text{ yd}^2$$

Test Practice

8. **Open Response** The exact circumference of a circle is 34π meters. What is the approximate area of the circle? Use 3.14 for π . Round to the nearest hundredth if necessary.

$$\frac{34\pi}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = 17$$

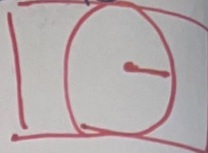
$$A = \pi r^2 = 3.14 \times 17 \times 17$$

$$= 907.46 \text{ m}^2$$

$L = \text{side of square} = \sqrt{81} = 9$ $r = \frac{1}{2}L$ $\frac{9}{2} = 4.5$

Apply

9. Tye has a square piece of yellow felt that has an area of 81 square inches. She wants to cut the largest circle possible from the material to create a sun for her art project. What is the area of the felt circle? Use 3.14 for π . Round to the nearest hundredth if necessary.



$A = \pi r^2$ $3.14 \times 4.5 \times 4.5 = 63.5925$

10. Tarek has 72 feet of plastic fencing to make a flower garden in his backyard. The garden shape can either be circular or square. If he uses all of the fencing, what is the difference between the area of the circular garden and the square garden? Use 3.14 for π . Round to the nearest hundredth if necessary.

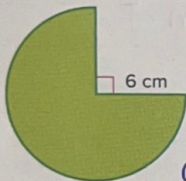
$P = 2\pi r$ ~~$P = 72$~~ $\frac{72}{2} = \frac{2\pi r}{2}$

$72 = 2\pi r$ $72 = 2 \times 3.14 \times r = \frac{72 \times 6.28}{6.28} = \frac{452.16}{6.28}$ $r = 11.47$
 $A = \pi r^2$ $3.14 \times 11.47 \times 11.47 = 413.16$

P of square = 72

$\frac{49}{4} = \frac{72}{4}$ $a = 18$ $A = (18)^2 = 324$
 $413.16 - 324 = 89.16$

11. **MP Reason Inductively** Explain how you could find the area of the three-quarter circle shown. Then write a formula that could be used to find the area of the three-quarter circle and use the formula to find the area of the figure. Use 3.14 for π .

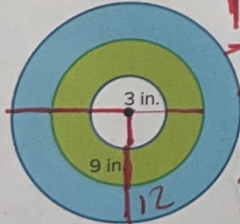


To find area multiply the area of circle

$A = \frac{3}{4} \pi r^2$ by $\frac{3}{4}$
 $\frac{3}{4} \times 3.14 \times 6 \times 6 = 84.78 \text{ cm}^2$

13. **MP Persevere with Problems** The bullseye on an archery target has a radius of 3 inches. The entire target has a radius of 9 inches. To the nearest hundredth, find the area of the target outside of the bullseye. Use 3.14 for π .

14. **MP Justify Conclusions** Determine if the following statement is true or false. Support your answer with an example or counterexample.



~~$\pi(9^2) - \pi(3^2)$~~ bullseye
 $\pi(9^2) - \pi(3^2)$
 $\pi(81 - 9)$
 $3.14 \times (81 - 9)$
 $3.14 \times 72 = 226.08 \text{ in}^2$

If the length of a radius is doubled, the area of the circle is also doubled.

No, Area is 4 times as great