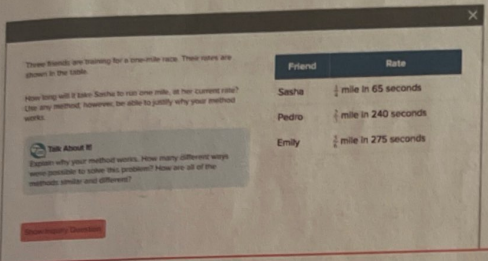
Unit Rates Involving Ratios of Fractions

I Can... find unit rates when one or both quantities are fractions.

What Vocabulary Will You Learn?
unit rate

Explore Find Unit Rates with Fractions

Online Activity You will use bar diagrams to explore how to find a unit rate when one or both quantities of a given rate are fractions.



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Learn Unit Rates Involving Ratios of Fractions

A recipe to make a diluted cleaning solution calls for 4 gallons of water mixed with $\frac{1}{3}$ cup of cleaner. The ratio of gallons of water to cups of cleaner is 4 to $\frac{1}{3}$ or $4 : \frac{1}{3}$. You have 1 cup of cleaner and you want to use all of it. How many gallons of water do you need to mix with 1 cup of cleaner to maintain the ratio $4 : \frac{1}{3}$?

To find the *unit rate*, the number of gallons of water needed to mix with 1 cup of cleaner, you can use various strategies.

Study Tip

You learned about ratios, rates, and unit rates in a previous grade.

A *ratio* is a comparison of two quantities, in which for every a units of one quantity, there are b units of another quantity.

A *rate* is a ratio that compares two quantities with unlike units.

A *unit rate* compares the first quantity per every 1 unit of the second quantity.

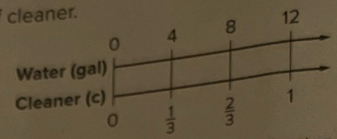
Your Notes

Talk About It!
Explain to a partner why the rate $4 : \frac{1}{3}$ is not a unit rate.

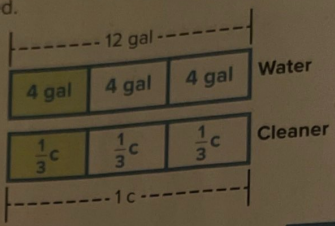
Study Tip
The expression $\frac{4}{\frac{1}{3}}$ is called a *complex fraction*. A complex fraction is a fraction in which the numerator or denominator, or both, are also fractions.

Talk About It!
How is finding a unit rate when one of the quantities is a fraction similar to finding a unit rate when both quantities are whole numbers? How is it different?

The double number line shows that, for every $\frac{1}{3}$ cup of cleaner, 4 gallons of water are needed. So, 12 gallons of water are needed to mix with 1 cup of cleaner.



The bar diagram also shows that 12 gallons of water are needed to mix with 1 cup of cleaner. For every $\frac{1}{3}$ cup of cleaner, 4 gallons of water are needed.



By creating a ratio table, you can scale forward by multiplying both $\frac{1}{3}$ and 4 by 3. The ratio table confirms that 12 gallons of water are needed to mix with 1 cup of cleaner.

Cleaner (c)	$\frac{1}{3}$	1
Water (gal)	4	12

×3 ×3

You can also use division when finding a unit rate. Recall that a ratio or rate can be written in fraction form. The ratio 4 : 1 can be written in fraction form as $\frac{4}{1}$. The ratio $4 : \frac{1}{3}$ can be written in fraction form as $\frac{4}{\frac{1}{3}}$.

Because a fraction bar indicates division, you can divide the numerator by the denominator to find the unit rate.

$\frac{4}{\frac{1}{3}} = 4 \div \frac{1}{3}$ The fraction bar indicates division.

$= \frac{4}{1} \div \frac{1}{3}$ Write 4 as $\frac{4}{1}$.

$= \frac{4}{1} \cdot \frac{3}{1}$ Multiply by the multiplicative inverse of $\frac{1}{3}$.

$= \frac{12}{1}$, or 12 Multiply the fractions.

So, $\frac{4}{\frac{1}{3}} = 12$.

Using any of these strategies, the unit rate is 12 gallons of water for every 1 cup of cleaner.

Example 1 Find Unit Rates

Tia is painting one side of her shed. She paints 36 square feet in 45 minutes.

At this rate, how many square feet can she paint each hour?

You know that 45 minutes is $\frac{3}{4}$ of an hour. So, Tia's rate is 36 square feet per $\frac{3}{4}$ hour. You need to find the unit rate, the number of square feet she can paint per 1 hour.

sq ft → 36 → ?
min → 45 → 60

per = unit = 1

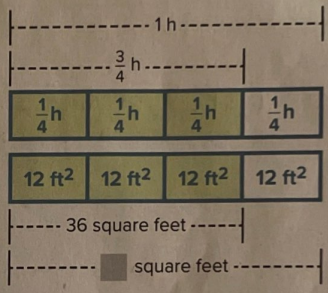
$\frac{36 \times 60}{45} = \frac{2160}{45}$

$= 48$

Method 1 Use a bar diagram.

Draw two bars to model the ratio 36 : $\frac{3}{4}$. Divide each bar into four sections, because $\frac{3}{4}$ is a multiple of $\frac{1}{4}$, and there are 4 sections of $\frac{1}{4}$ hour in 1 hour.

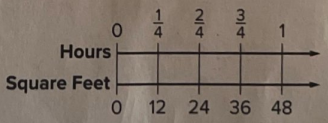
To find the unit rate, first find the value of each section in the bar representing square feet. Because three sections have a value of 36 ÷ 3, or 12 square feet. Because 4(12) = 48, the unit rate is 48 square feet per hour.



Method 2 Use a double number line.

The top number line represents the number of hours. The bottom number line represents the number of square feet. Mark and label the ratio 36 : $\frac{3}{4}$.

Mark and label four equal increments of $\frac{1}{4}$ on the top number line. Mark the same number of equal increments on the bottom number line.



Each increment on the bottom number line represents 12 square feet. Because 4(12) = 48, the unit rate is 48 square feet per hour.

Think About It!
Why is 45 minutes equal to $\frac{3}{4}$ of an hour?

Talk About It!
Use mathematical reasoning to explain why Tia can paint more than 36 square feet per hour.

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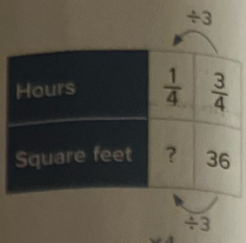
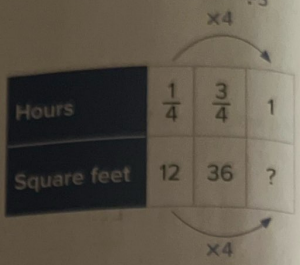
Talk About It!
Why do you need to scale backward first before scaling forward?

Method 3 Use a ratio table.

The ratio table shows the number of square feet painted in $\frac{3}{4}$ hour. Scale backward to find the number of square feet painted in $\frac{1}{4}$ hour.



Scale forward to find the number of square feet Tia can paint in 1 hour. This is the unit rate.



Because $\frac{1}{4}(4) = 1$, multiply 12(4). So, Tia can paint 12(4), or 48 square feet, in one hour.

Talk About It!
Compare the four methods. What operation(s) did you use with each of the methods?

Method 4 Use division.

The rate 36 square feet in $\frac{3}{4}$ hour can be written as $\frac{36}{\frac{3}{4}}$.

$\frac{36}{\frac{3}{4}} = 36 \div \frac{3}{4}$ Write the complex fraction as a division problem.

$= \frac{36}{1} \div \frac{3}{4}$ Write 36 as $\frac{36}{1}$.

$= \frac{36}{1} \cdot \frac{4}{3}$ Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$= \frac{144}{3}$ or 48 Multiply. The unit rate is 48 ft^2 per hour.

So, Tia can paint 48 square feet in one hour.

Check

Doug entered a canoe race. He paddled 5 miles in $\frac{2}{3}$ hour. What is his average speed in miles per hour? Use any strategy.

Show your work here.

mile hour
5 $\frac{2}{3}$
? 1

$5 \div \frac{2}{3}$

$\frac{5 \times 3}{2} = \frac{15}{2} = 7\frac{1}{2}$

Go Online You can complete an Extra Example online.

Example 2 Find Unit Rates

Josiah can jog $\frac{5}{6}$ mile in 15 minutes.

Find his average speed in miles per hour.

You know that 15 minutes is $\frac{1}{4}$ hour. So, Josiah's rate is $\frac{5}{6}$ mile per $\frac{1}{4}$ hour. You need to find the unit rate, the number of miles he can jog per 1 hour.

Think About It!
What do you notice about both quantities of the rate?

1hr = 60min

miles / min

$\frac{5}{6}$ / 15

?

60

$\frac{5}{6} \times \frac{60}{15}$

$\frac{5}{6} \times \frac{10}{1}$

$50 \div 15 =$

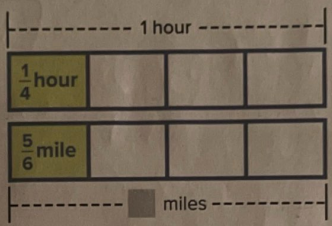
$\frac{50 \div 5}{15 \div 5} = \frac{10}{3}$

or

$3\frac{1}{3}$

Method 1 Use a bar diagram.

Draw two bars to model the ratio $\frac{5}{6} : \frac{1}{4}$. Divide each bar into 4 sections, because there are 4 sections of $\frac{1}{4}$ in 1 hour.

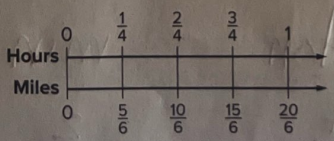


To find the unit rate, first find the value of each section in the bar representing miles. Each section has a value of $\frac{5}{6}$ mile. Because $4\left(\frac{5}{6}\right) = \frac{20}{6}$ or $3\frac{1}{3}$, the unit rate is $3\frac{1}{3}$ miles per hour.

Method 2 Use a double number line.

The top number line represents the number of hours. The bottom number line represents the number of miles. Mark and label the ratio $\frac{5}{6} : \frac{1}{4}$.

Mark and label four equal increments of $\frac{1}{4}$ on the top number line. Mark the same number of equal increments on the bottom number line.



Each increment on the bottom number line represents $\frac{5}{6}$ mile.

Because $4\left(\frac{5}{6}\right) = \frac{20}{6}$, or $3\frac{1}{3}$, the unit rate is $3\frac{1}{3}$ miles per hour.

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Method 3 Use a ratio table.

The ratio table shows the number of miles jogged in $\frac{1}{4}$ hour. Scale forward to find the number of miles Josiah can jog in 1 hour. This is the unit rate.

Hours	$\frac{1}{4}$	1
Miles	$\frac{5}{6}$	?

×4 ×4

Because $\frac{1}{4}(4) = 1$, multiply $\frac{5}{6}(4)$. So, Josiah can jog $\frac{5}{6}(4)$, or $3\frac{1}{3}$ miles in one hour.

Talk About It!
How could you word this problem using minutes instead of hours? Does it change the outcome of Josiah's average speed?

Method 4 Use division.

The rate $\frac{5}{6}$ mile per $\frac{1}{4}$ hour can be written as $\frac{\frac{5}{6}}{\frac{1}{4}}$.

$\frac{\frac{5}{6}}{\frac{1}{4}} = \frac{5}{6} \div \frac{1}{4}$ Write the complex fraction as a division expression.

$= \frac{5}{6} \cdot \frac{4}{1}$ Rewrite division as multiplication.

$= \frac{20}{6}$ Multiply.

$= 3\frac{1}{3}$ Simplify. The unit rate is $3\frac{1}{3}$ miles per hour.

With each representation, because there are 4 quarter-hours in one hour, his average speed is found by multiplying by 4. So, Josiah can jog $3\frac{1}{3}$ miles per hour.

Check

A garden hose was left on in a yard and spilled $\frac{7}{8}$ gallon every $\frac{2}{3}$ minute. Find the average number of gallons spilled per minute. Use any strategy.

KCF
$\frac{7}{8} \div \frac{2}{3}$
$\frac{7}{8} \times \frac{3}{2} = \frac{21}{16}$
$= 1\frac{5}{16}$

gallon
$\frac{7}{8}$
?
1

min
$\frac{2}{3}$

Go Online You can complete an Extra Example online.

Check

A walk-a-thon was held at a local middle school. The table gives the average distances and times for four walkers for certain periods of the walk-a-thon. If the total distance of the route was 2 miles, who completed the route first? How long did it take her to complete the route if she walked at a constant rate?

Person	Distance (mi)	Time (min)
Lakeisha	$\frac{7}{8}$	18
Baydan	$\frac{9}{10}$	22
Madison	$\frac{4}{5}$	17

Lakeisha 2 mi / $\frac{7}{8}$ × 18 min = 41.4

Baydan $\frac{9}{10}$ × 22 = 48.8

Madison $\frac{4}{5}$ × 17 = 42.5

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare the process for finding unit rates involving fractions with what you know about dividing fractions. How are they similar? How are they different?

So, Lakeisha complet the Route finst

Name _____ Period MS / AVA

Practice

Go Online You can complete your homework online.

Solve each problem. Use any strategy, such as a bar diagram, double number line, ratio table, or division.

1. A truck driver drove 48 miles in 45 minutes. At this rate, how many miles can the truck driver drive in one hour? (Example 1)

2. Russell runs $\frac{9}{10}$ mile in 5 minutes. At this rate, how many miles can he run in one minute? (Example 1)

3. A small airplane flew 104 miles in 50 minutes. At this rate, how many miles can it fly in one hour? *(50 minutes = $\frac{5}{6}$ hour)* (Example 1)

4. DeAndre downloaded 8 apps onto his tablet in 12 seconds. At this rate, how many apps could he download in one minute? *(12 seconds = $\frac{1}{5}$ minute)* (Example 1)

5. In Lixue's garden, the green pepper plants grew 5 inches in $\frac{3}{4}$ month. At this rate, how many feet can they grow in one month? *(Let 5 inches = $\frac{5}{9}$ foot)* (Example 2)

6. Thunder from a bolt of lightning travels $\frac{1}{10}$ mile in $\frac{1}{2}$ second. At this rate, how many miles can it travel in one second? (Example 2)

7. The average sneeze can travel $\frac{3}{100}$ mile in 3 seconds. At this rate, how far can it travel in one minute? *(3 seconds = $\frac{1}{20}$ minute)* (Example 2)

Test Practice

8. **Multiselect** Anita is making headbands for her softball team. She needs a total of $\frac{3}{4}$ yard of fabric. Select all types of fabric that cost less than $8 per yard. (Example 2)

- [x] cotton
- [x] flannel
- [x] fleece
- [ ] terry cloth

Fabric	Total Cost for $\frac{3}{4}$ Yard ($)
Cotton	5.54
Flannel	2.62
Fleece	4.27
Terry Cloth	6.52

Apply

9. During the first seconds after takeoff, a rocket traveled 208 kilometers in 50 minutes at a constant rate. Suppose a penny is dropped from a skyscraper and could travel 153 kilometers in $\frac{1}{2}$ hour at a constant rate. Which of these objects has a faster unit rate per hour? How much faster?

10. To prepare for a downhill skiing competition, Roman completed three training sessions. The table shows his average time and distance for each session. Did Roman's rate, in miles per hour, increase from session to session? Write an argument that can be used to defend your solution.

Session	Time (hr)	Distance (mi)
1	$\frac{2}{125}$	$\frac{9}{10}$
2	$\frac{3}{200}$	$\frac{11}{12}$
3	$\frac{3}{250}$	$\frac{17}{20}$

11. **Reason Abstractly** Explain why a student who runs $\frac{3}{4}$ mile in 6 minutes is faster than a student who runs $\frac{1}{2}$ mile in 5 minutes.

12. Compare and contrast the rates $\frac{4}{5}$ *mile in 8 minutes* and *4 minutes to travel* $\frac{2}{5}$ *mile.*

13. **Find the Error** Carli made 9 greeting cards in $\frac{3}{4}$ hour. She determined her unit rate to be $\frac{1}{12}$ card per hour. Find her error and correct it.

14. **Be Precise** A standard shower drain can drain water at the rate of 480 gallons in $\frac{2}{3}$ hour. Create three different rates, using the same or different units, that are all equivalent to this rate. Be precise in the units you choose. Then find the unit rate, in gallons per minute.